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LETTER TO THE EDITOR

The dynamic critical exponent of the $q = 3$ and 4 state Potts model

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Abstract. The dynamic Monte Carlo renormalisation group method introduced by Jan, Moseley and Stauffer is used to determine the dynamic critical exponent, z, for the $q = 3$ and $q = 4$ state Potts model in two dimensions. We find that $z = 2.43 \pm 0.15$ and 2.36 ± 0.20 for the $q = 3$ and $q = 4$ state Potts models. These results are in disagreement with the recent conjecture of Domany.

The static critical properties of the two-dimensional $q = 1, 2, 3$ and 4 state Potts models are well known (see, e.g., Wu (1982) for a recent review). However, there is very little information about the dynamical properties of these models either with Glauber or Kawasaki dynamics. Forgacs *et al* (1980) extended a Migdal-type real space renormalisation method to analyse the dynamical properties and within a totally self-consistent calculation reported $z = 1.82$ ($q = 2$), 1.92 ($q = 3$) and 2.0 ($q = 4$). Binder (1981) measured the non-linear relaxation exponent, Δ^{nl}, of the magnetisation for $q = 2$, 3 and 4 and found the Δ^{nl} was more or less independent of q and ~ 1.9. The values for z obtained through the scaling relation $z = (\Delta^{nl} + \beta)/\nu$ are 2.03 ($q = 2$), 2.41($q = 3$) and 2.98 ($q = 4$). Recently Tang and Landau (1986) have initiated a detailed re-examination of the non-linear exponent of the Potts model and have also found that Δ^{nl} is independent of q and ~ 2.1 for $q = 2$, 3 and 4. Tobochnik and Jayaprakash (1982), using a dynamic Monte Carlo renormalisation group, found z ($= 2.70 \pm 0.4$) for the $q = 3$ state Potts model on the square lattice. Finally Aydin and Yalabik (1984, 1985) presented two calculations of z for the $q = 3$ model: (i) a dynamic Monte Carlo renormalisation group ($z = 2.7 \pm 0.4$) and (ii) a finite-size scaling analysis ($z = 2.2 \pm 0.1$). Domany (1984) mapped the dynamic properties of the two-dimensional Potts model onto a two-dimensional cellular automata model which in turn was mapped onto a static equilibrium three-dimensional Ising model. The decay of the correlation function in the direction perpendicular to the original two-dimensional plane together with a particular, though arbitrary, choice of the singular behaviour of the free energy exponent allowed for the determination of the critical exponent $z = 2$ ($q = 2$) with possible logarithmic corrections 2.8 ($q = 3$) and 4.0 ($q = 4$). Kalle's (1984) work on the two-dimensional Ising model has cast some doubt on this conjecture.

In this letter the dynamic exponent for the two-dimensional $q = 3$ and 4 state Potts model is determined. The dynamic Monte Carlo renormalisation group approach of Jan *et al* (1983) with Glauber dynamics has been successfully applied to the 2D and

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3D Ising models (Kalle 1984) and moreover yielded exact results for the one-dimensional $q = 2$, 3 and 4 state Potts model (Leyvraz and Jan 1986). This method will be used to determine the dynamic properties of the two-dimensional Potts model.

Finite-size scaling arguments predict that in a system of linear extent L, the equilibrium magnetisation M obeys the following ansatz:

$$M_L = L^{-\beta/\nu} f[(T - T_c)L^{1/\nu}] \tag{1}$$

as the time t tends to ∞. T represents the temperature, β the order parameter exponent and ν the correlation length exponent.

The non-equilibrium relaxation of the magnetisation is described by

$$M_L = L^{-\beta/\nu} f[(T - T_c)L^{1/\nu}, t/L^z] \tag{2}$$

and at $T = T_c$, the critical temperature, by

$$M_L = L^{-\beta/\nu} g(t/L^z). \tag{3}$$

Thus the magnetisation, M_L, of a system of size L_1 at time t_1 will be equal at t_2 to the magnetisation M_{L_2} of a system of size L_2 if we have

$$\frac{t_1}{t_2} = \left(\frac{L_1}{L_2}\right)^{-\beta z/\nu}. \tag{4}$$

Let us now consider a system and its renormalised images formed by the blocking of spins in a cell of size b. In a system of size L there are $(L/b)^d$ superspins where each superspin is ± 1 depending on the majority of spin orientations within a cell of size b. Stauffer (1984) has shown that the magnetisation of the renormalised system is described by

$$M_b \sim h(t/b^z) \qquad \text{at } T = T_c. \tag{5}$$

Thus when $M_{b_1} = M_{b_2}$ the times t_1 and t_2 are related by

$$\frac{t_1}{t_2} = \left(\frac{b_1}{b_2}\right)^z. \tag{6}$$

It is important to emphasise that no prefactor $b^{-\beta/\nu}$ enters this expression.

We consider a 2D system of Potts spins whose energy is described by

$$H = -J \sum_{\sigma_i \sigma_j} \delta_{\sigma_i \sigma_j} \qquad \text{where } \sigma_i = 1, 2, \ldots, q.$$

By means of the scaling relation (6) we are able to calculate z through the following procedure. The system is initialised in a ground state and allowed to evolve to equilibrium configurations at T_c through Glauber dynamics. The Monte Carlo (Binder 1984) procedure consists of visiting the sites in a sequential manner and flipping the spin from its present state with a probability P_n proportional to the local configuration, i.e.

$$P_n = \begin{cases} 1 & \Delta E \leq 0 \\ \exp(-\Delta E/kT_c) & \Delta E > 0 \end{cases} \qquad \Delta E = E_n - E_p$$

where P_n is the probability of the new state, E_n the energy of the new state if the spin flip is successful and E_p the energy of the present state. The Monte Carlo routine was written in multi-spin coding form (Rebbi and Swendsen 1980) and this allows the simulation of lattice systems of $L = 800$ for $q = 3$ and 4. The renormalisation factors

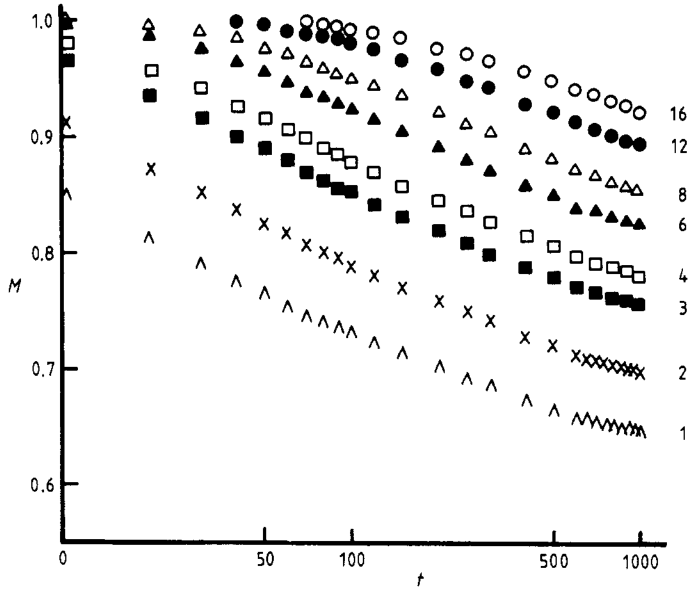

Figure 1. Variation of the original magnetisation and the renormalised magnetisation with time (Monte Carlo steps per spin) for the $q = 3$ Potts model. The numbers on the data sets give b, the length rescaling factor.

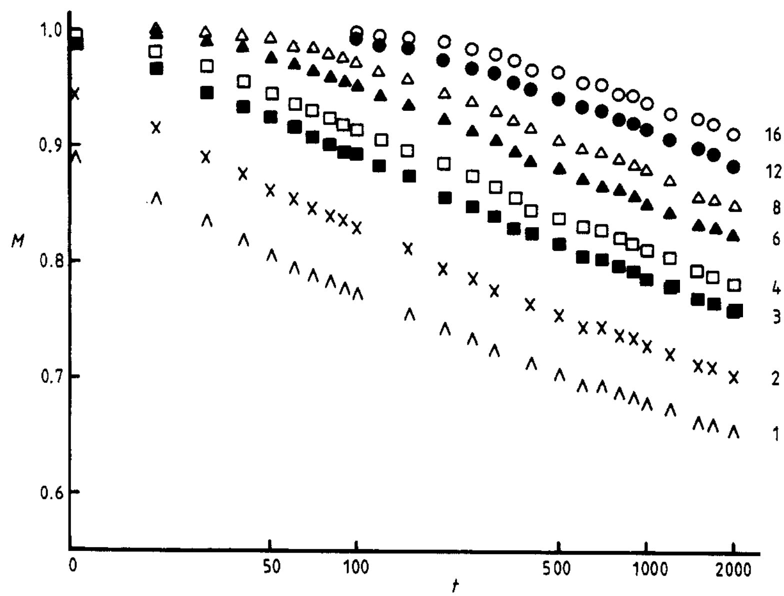

Figure 2. Variation of the original magnetisation and renormalised images with time (Monte Carlo time steps per spin) for the $q = 4$ Potts model. The numbers on the data sets give b, the length rescaling factor.

for the cells are $b = 2, 3, 4, 6, 8, 12$ and 16. The order parameter for the system and the renormalised images are shown in figure 1 for $q = 3$ (average over 12 trials) and t up to 1000 Monte Carlo time steps. Figure 2 shows the equivalent data for $q = 4$ where the averages are over 8 trials and $t = 2000$ Monte Carlo time steps. The analysis of the data proceeds as follows. The timescale is shifted by an amount $\Delta t = t - t'$ in order to observe maximum overlap between the magnetisation of the different renormalised systems for the same value of the ratio of b'/b. The dynamic exponent z is then defined as $\ln \Delta t / \ln(b'/b)$ and is 2.43 ± 0.15 for $q = 3$ and 2.36 ± 0.20 for $q = 4$. We also re-analysed the $q = 2$ Potts model to find $z = 2.19 \pm 0.1$ in excellent agreement with other numerical values (see table 1).

Table 1. The shifts along the time axis necessary to produce maximum overlap in the magnetisation for different scale factors.

b'/b	$q = 2$	$q = 3$	$q = 4$
	Displacement of graphs $t'/t(z)$		
2	4.5 (2.17)	5.2 (2.38)	5.2 (2.38)
3	11.5 (2.22)	14 (2.40)	12.0 (2.26)
4	22.5 (2.25)	24 (2.29)	22 (2.23)
$\frac{3}{2}$	2.4 (2.16)	2.9 (2.63)	2.8 (2.54)
$\frac{4}{3}$	1.85 (2.14)	2.0 (2.40)	1.95 (2.34)
Average z	$\sim 2.19 \pm 0.05$	2.43 ± 0.15	2.36 ± 0.2

The dynamic exponents found for the $q = 3$ and 4 Potts model clearly exclude the conjecture of Domany (1984). Our results are in agreement with the self-consistent Migdal dynamic renormalisation work of Forgacs *et al* (1980) and also with the $q = 3$ finite-size scaling approach of Aydin and Yalabik (1985), Tang and Landau (1986), Tobochnik and Jayaprakash (1982) and Binder (1981), but not the others referred to earlier. The results obtained for the $q = 4$ model are in marked disagreement with the Monte Carlo results. It is possible that the system may still, at $t = 2000$ Monte Carlo time steps, be too far from equilibrium; however, this can only be resolved with a ten-fold increase in CPU time (~ 100 h on an IBM 3091). Our results clearly indicate that z is only weakly dependent on q for $q \geqslant 2$.

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